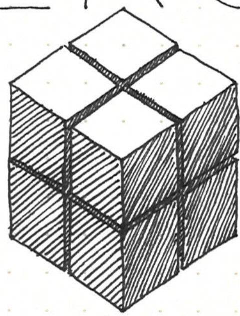
Testing & Learning Convex Sets in the Ternary Hypercube

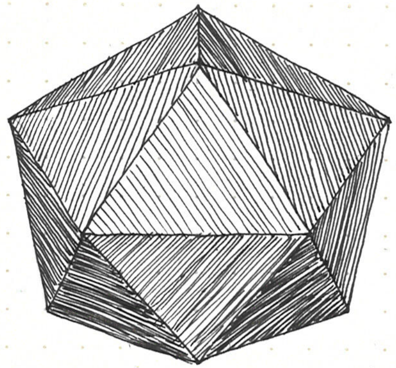


Hadley Black

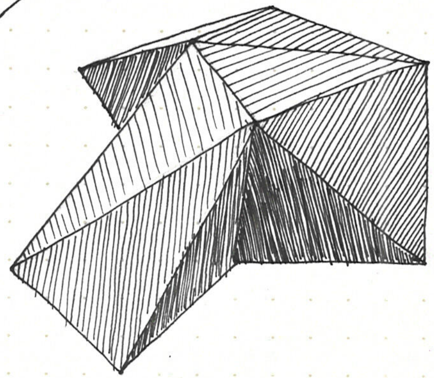
Eric Blais

Nathan Harms

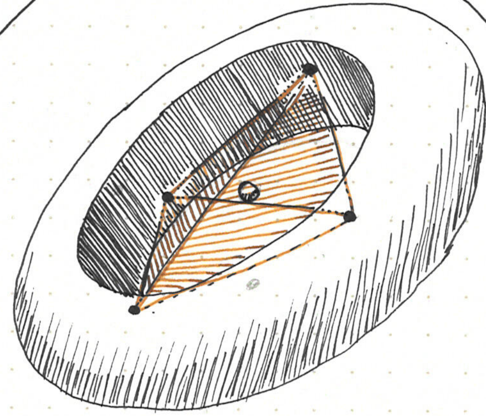
Testing Convex Sets



Convex



ε-Far from Convex

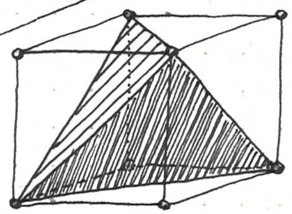


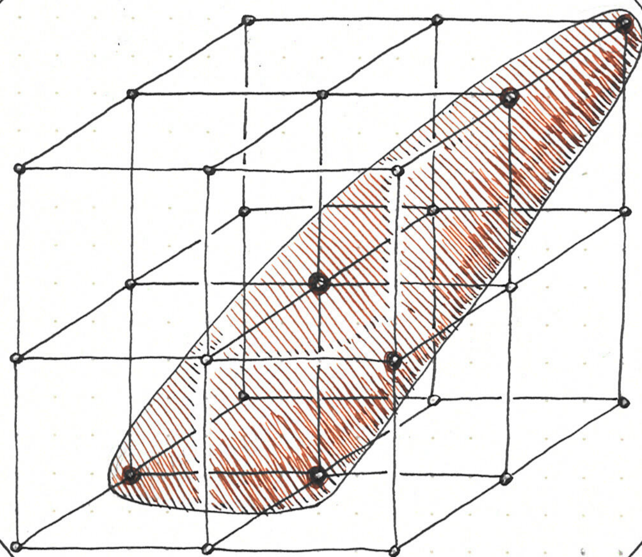
Tight bounds:

- Low dimension [BMR'16 × 3, R'03]
- Gaussian over $\mathbb{R}^n$ [CFSS'17, KOS'08]
  - ↳ samples only

Unclear how to use queries [BB'20, RV'04]

Discrete convex sets?

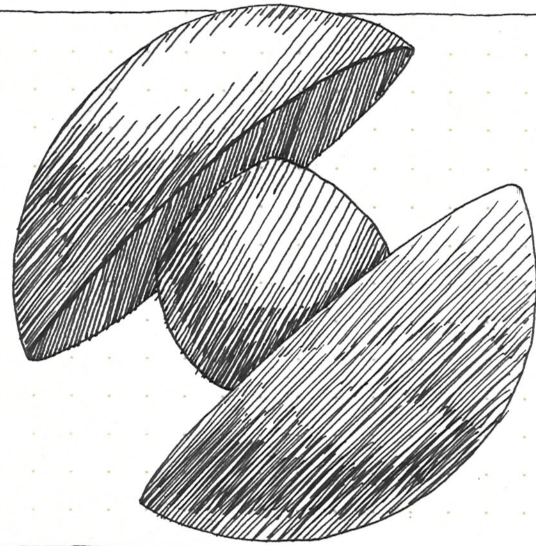


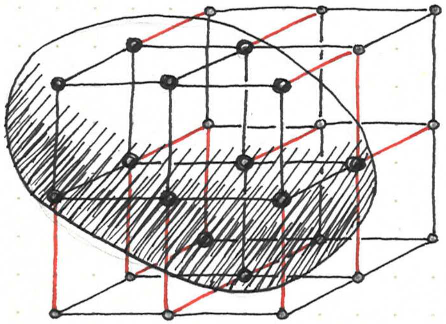


Theorems

Non-Adaptive Testing, One-Sided Error

- Upper bound: $3^{\tilde{O}(\sqrt{n})}$
- Lower bound: $3^{\Omega(\sqrt{n})}$



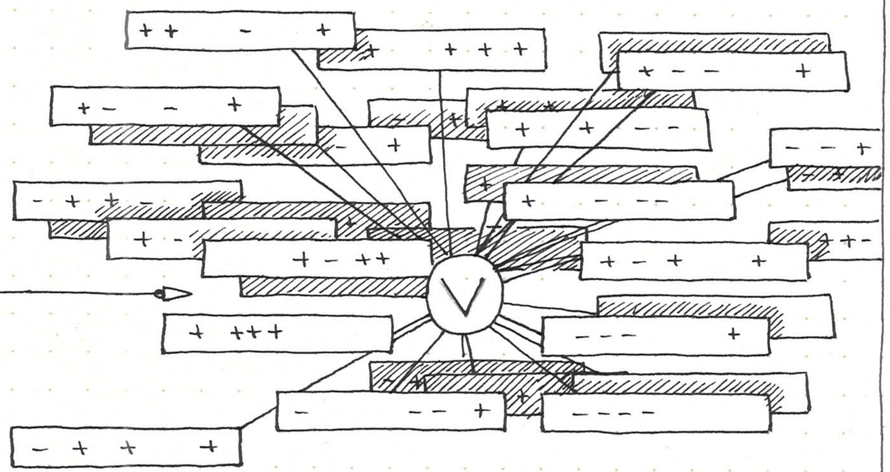


Edge Boundary $(2n \cdot 3^{n-1}$ edges total$)$

- Convex sets cut $\tilde{O}(n^{3/4}) \cdot 3^{n-1}$ edges
- Convex sets can cut $\Omega(n^{3/4}) \cdot 3^{n-1}$ edges

Sample-Based Testing & Learning

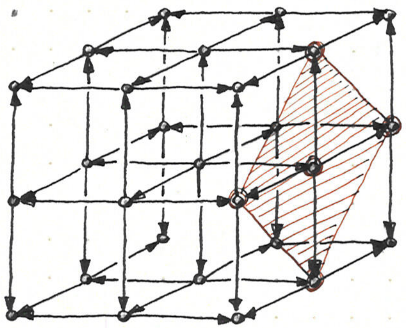
- One-sided error lower bound: $3^{\Omega(n)}$
- Two-sided error lower bound: $3^{\Omega(\sqrt{n})}$
- Two-sided error upper bound: $3^{\tilde{O}(n^{3/4})}$



Talagrand's random DNFs [BB16, CWX17]

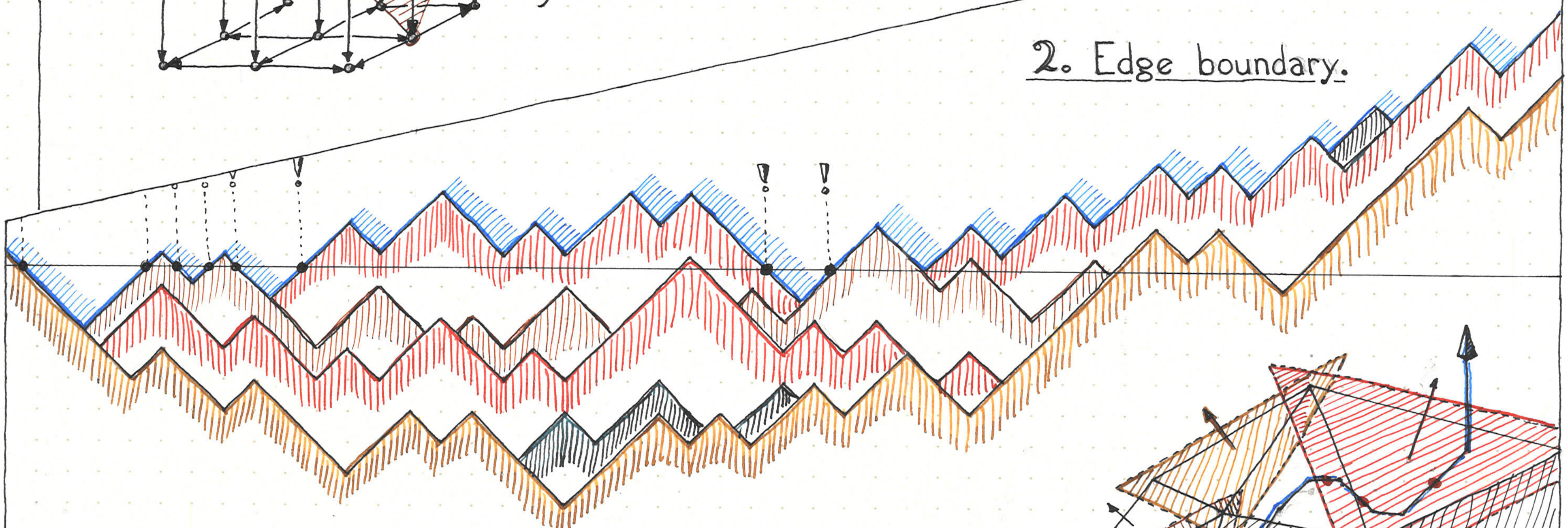
Upper Bounds

1. Non-adaptive testing.



Look up! ...but not too far.

2. Edge boundary.



$\mathbb{E}[\#\text{ crossings of } m\text{-step "max-walk"}] = O(\sqrt{m})$